



Universidad
Rey Juan Carlos

Escuela Técnica Superior
Ingeniería Informática

Teach the importance of logic (programming)
in Computer Science and why it is important.

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Objetivo

Support the importance of teaching logic (and logic programming) in computer science degrees:

- Logic allows us to model (almost) everything and to reason about it.
- Some historical milestones:
 - 0, 1, ... Human reasoning: 4th century BC, 13th and 17th century, etc.
[...]
 - n Quantum mechanics: 21st century.
 - $n + 1$ (A lot of) other things.

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 - $m + h$ In 1920, Hilbert proposed to axiomatize mathematics...
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 - \dots, m, \dots Mathematics: mainly from 1910 to 1930.
 - $m + h$ In 1920, Hilbert proposed to axiomatize mathematics...
 - $m + g$... but in 1931, Gödel shows that it is not possible.
 - n Quantum mechanics: 21st century.
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Incompleteness Theorem (Gödel, 1931)

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Second incompleteness Theorem (Gödel, 1931)

Particular case of the first Theorem: One of the undecidable statements of a theory is the one that affirms the consistency of the theory.

... and also the computation inherits these these limitations

Answer to *Entscheidungsproblem* (Church, 1936)

First-order logic does not have a decision procedure (algorithm) that allows us to prove that any formula is a theorem of this logic. This logic is therefore considered **undecidable**.

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Given a Turing machine M and a word w , determine whether M terminates in a finite number of steps when executed using w as input, is **undecidable**.

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Given a Turing machine M and a word w , determine whether M terminates in a finite number of steps when executed using w as input, is **undecidable**.

- Logic and Computation **are equivalent**: Determining the halting problem is reduced to proving in FOL the formula that expresses the existence of an output from the application of a series of instructions. **PLAY¹**

Ver "Las Matemáticas tienen una Terrible Falla" (<https://bit.ly/3sFyaxq>)

Who I am



Joaquín Arias

Professor at Universidad
Rey Juan Carlos (URJC).

- 2020 - today: Researcher at Group of IA at CETINIA, URJC.
- 2013 - 2020: Researcher at IMDEA Software Institute.
- Academic background:
 - Ph.D. in Computer Science (2020).
 - M.Sc. in Software and Systems (2015).
 - B.Sc. in Mathematics and informatic (2014).
 - M.Arch. in Architecture (2002).
- PhD Thesis: "Advanced Evaluation Techniques for (Non)-Monotonic Reasoning using Rules with Constraints".

Motivation and some teaching experience at URJC

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- Third and fourth year: The acquired knowledge of logic is not valued.
 - ...and there are deficiencies writing technical reports (including analysis or synthesis) and in preparing and giving presentations.
- Final Degree Project: Last opportunity to incorporate issues related to logic, so important in the professional development of an engineer:
 - Motivation of the importance of the problem to be solved,
 - Modeling of problems; definition of objectives; analysis of existing solutions, justification of the proposal; synthesis of the results; etc.

Motivation and some teaching experience at URJC (cont.)

- Linking Logic With Programming

Motivation and some teaching experience at URJC (cont.)

- Linking Logic With Programming
 - Natural Deduction + DeduccionNatural.pl
 - How teach development techniques used in the industry.
 - E.g., in groups choose, study and present one of them in depth.
 - Present historical context of Logic and its limitations:
 - E.g., there are activities that cannot be automated.
 - ... computer scientists' work is one of these activities.
 - Actual objective of these activities:
 - Awake critical thinking.
 - E.g., searching for information (also in research articles).
 - ... writing a short report with writing standards (and \LaTeX).
 - ... and/or preparing and giving a presentation.

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 - Actual objective of these activities:
 - Awake critical thinking (**and interest in research**).
E.g., searching for information (also in research articles).
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One Example: Natural deduction + DeduccionNatural.pl

- Propositional logic is simple and its semantics is sound and complete.
- First order logic represents knowledge more clearly and concisely.
- We infer new knowledge from premises: [Aristotle IV BC]
All men are mortal. Socrates is a man. Therefore Socrates is mortal.

First order logic:

$$\forall x (Man(x) \rightarrow Mortal(x))$$

$$Man(socrates)$$

$$Mortal(socrates)$$

Horn clauses:

$$Mortal(x) \vee \neg Man(x)$$

$$Man(socrates)$$

$$\neg Mortal(socrates)$$

Prolog:

```
1 mortal(X) :- man(X).
```

```
2 man(socrates).
```


```
3
```

```
4 ?- mortal(socrates).
```

- A subset of the first order logic is decidable [Horn 1951, Robinson 1965].

One Example: Natural deduction + DeduccionNatural.pl

Demonstrations in natural deduction as programs

- Using DeduccionNatural.pl [Arias et al. 2023].
 - Implemented using Prolog, a logic programming language.
 - Can be executed online. 
- Based on the Natural Deduction method by Gentzen (see [Arias 2022; Gallinari 2009; Gentzen 1935] for details):
 - The inference rules are predefined functions.
 - The proofs are programs.
 - The derived rules are subroutines.
- The student can read DeduccionNatural.pl (and manipulate it).

Natural Deduction by Gentzen I

Introducción de \wedge (I_{\wedge})

$$\frac{\begin{array}{c} A \\ B \end{array}}{A \wedge B}$$

$\top[p, q] \vdash p \wedge q$	
1. p	premisa
2. q	premisa
3. $p \wedge q$	I_{\wedge} (1,2)

Eliminación de \wedge (E_{\wedge})

$$\frac{A \wedge B}{A} \qquad \frac{A \wedge B}{B}$$

$\top[p \wedge q] \vdash p$	
1. $p \wedge q$	premisa
2. p	E_{\wedge} (1)

$\top[p \wedge q] \vdash q$	
1. $p \wedge q$	premisa
2. q	E_{\wedge} (1)

Natural Deduction by Gentzen II

Eliminación de \vee (E_{\vee})

$$\frac{\begin{array}{l} A \vee B \\ A \rightarrow C \\ B \rightarrow C \end{array}}{C}$$

$\top[p \vee q, p \rightarrow \neg r, q \rightarrow \neg r] \vdash \neg r$	
1. $p \vee q$	premisa
2. $p \rightarrow \neg r$	premisa
3. $q \rightarrow \neg r$	premisa
4. $\neg r$	$E_{\vee} (1,2,3)$

Introducción de \vee (I_{\vee})

$$\frac{A}{A \vee B} \quad \frac{B}{A \vee B}$$

$\top[p] \vdash p \vee r$	
1. p	premisa
2. $p \vee r$	$I_{\vee} (1)$

$\top[p] \vdash r \vee p$	
1. p	premisa
2. $r \vee p$	$I_{\vee} (1)$

Natural Deduction: Implement a simple example

$$T[s \wedge (p \vee q), p \rightarrow \neg r, q \rightarrow \neg r] \vdash s \wedge \neg r$$

Natural Deduction: Implement a simple example

$$T[s \wedge (p \vee q), p \rightarrow \neg r, q \rightarrow \neg r] \vdash s \wedge \neg r$$

1. $s \wedge (p \vee q)$ *premisa*
2. $p \vee q$ $E_{\wedge}(1)$
3. $p \rightarrow \neg r$ *premisa*
4. $q \rightarrow \neg r$ *premisa*
5. $\neg r$ $E_{\vee}(2, 3, 4)$
6. s $E_{\wedge}(1)$
7. $s \wedge \neg r$ $I_{\wedge}(5, 6)$

Natural Deduction: Implement a simple example

$$T[s \wedge (p \vee q), p \rightarrow \neg r, q \rightarrow \neg r] \vdash s \wedge \neg r$$

- | | | |
|----|------------------------|---------------------|
| 1. | $s \wedge (p \vee q)$ | <i>premisa</i> |
| 2. | $p \vee q$ | $E_{\wedge}(1)$ |
| 3. | $p \rightarrow \neg r$ | <i>premisa</i> |
| 4. | $q \rightarrow \neg r$ | <i>premisa</i> |
| 5. | $\neg r$ | $E_{\vee}(2, 3, 4)$ |
| 6. | s | $E_{\wedge}(1)$ |
| 7. | $s \wedge \neg r$ | $I_{\wedge}(5, 6)$ |

The screenshot shows a Prolog playground interface with two panes. The left pane contains the Prolog code, and the right pane shows the execution output.

```

1 ~~~~~
2 % (c) 2023 Joaquin Arias (URJC)
3 % Name: DeduccinNatural.pl
4 % Author: Joaquin Arias
5 % Date: 22 April 2023
6 % Purpose: Execute Natural Deduction Proofs
7 % LICENSE: Apache License 2.0
8 ~~~~~
9
10 % Operator precedence
11 :- op(200, fx, []).
12 :- op(400, xfy, [and, or]).
13 :- op(600, xfy, [=>, <=>]).
14
15 % Auxiliary precedence for !
16 % Used to define the inference rules
17 :- op(400, xfy, !).
18
19 % Examples
20 ejemplo1 :-
21     main[ s and p or q, p => ! r, q => ! r ],
22     q and ! r,
23     [ %Premisa(1),
24       %E' and b(1),
25       %Premisa(2),
26       %Premisa(3),
27       %E' or (2, 3, 4),
28       %E' and a(1),
29       %I' and (6, 5)
30     ],
31     !.
32
33 ejemplo2 :-
34     main[ ! p => q and ! q ],
35     p,
36     [ %Premisa(1),
37       %I' (1),
38       %E' ! (2)
39     ],
40     !.

```

The right pane shows the execution output:

```

?- use_module('dratt.pl').
yes
?- ejemplo1.
T[s and p or q,p=>!r,q=>!r] |- s and!r
          Premisa(1)
          E and b(1)
          Premisa(2)
          Premisa(3)
          E or(2,3,4)
          E and a(1)
          I and(6,5)
          ok
yes
?-

```

Demo



Natural Deduction by Gentzen III

Introducción \neg (I $_{\neg}$)

$$\frac{A \rightarrow B \wedge \neg B}{\neg A}$$

Eliminación \neg (E $_{\neg}$)

$$\frac{\neg\neg A}{A}$$

$\top[\neg p \rightarrow q \wedge \neg q] \vdash p$	
1. $\neg p \rightarrow q \wedge \neg q$	premisa
2. $\neg\neg p$	I$_{\neg}$ (1)
3. p	E$_{\neg}$ (2)

Natural Deduction by Gentzen IV

Eliminación \rightarrow (E_{\rightarrow})

$$\frac{A \rightarrow B \quad A}{B}$$

Introducción \rightarrow (I_{\rightarrow})

$$\frac{A \text{ (supuesto)} \quad B}{A \rightarrow B}$$

$\top[p \rightarrow \neg r, \neg r \rightarrow q, p] \vdash q$	
1. $p \rightarrow \neg r$	premisa
2. p	premisa
3. $\neg r$	$E_{\rightarrow} (1,2)$
4. $\neg r \rightarrow q$	premisa
5. q	$E_{\rightarrow} (3,4)$

$\top[p \rightarrow q, q \rightarrow r] \vdash p \rightarrow r$	
1. $p \rightarrow q$	premisa
2. $q \rightarrow r$	premisa
3. p	supuesto
4. q	$E_{\rightarrow} (1,3)$
5. r	$E_{\rightarrow} (2,4)$
6. $p \rightarrow r$	$I_{\rightarrow} (3,5)$

Natural Deduction by Gentzen V

Introducción \leftrightarrow (I_{\leftrightarrow})

$$\frac{A \rightarrow B \quad B \rightarrow A}{A \leftrightarrow B}$$

$\top[p \rightarrow \neg r, \neg r \rightarrow p] \vdash p \leftrightarrow \neg r$	
1. $p \rightarrow \neg r$	premisa
2. $\neg r \rightarrow p$	premisa
3. $p \leftrightarrow \neg r$	$I_{\leftrightarrow} (1,2)$

Eliminación \leftrightarrow (E_{\leftrightarrow})

$$\frac{A \leftrightarrow B}{A \rightarrow B} \quad \frac{A \leftrightarrow B}{B \rightarrow A}$$

$\top[p \leftrightarrow q \wedge r, p] \vdash r$	
1. $p \leftrightarrow q \wedge r$	premisa
2. $p \rightarrow q \wedge r$	$E_{\rightarrow} (1)$
3. p	premisa
4. $q \wedge r$	$E_{\rightarrow} (2,3)$
5. r	$E_{\wedge} (4)$

Natural Deduction: Implement derived rules

$$\frac{A \rightarrow B \quad \neg B}{\neg A} \quad \textit{Modus Tollens (MT)}$$

Natural Deduction: Implement derived rules

$$\frac{A \rightarrow B \quad \neg B}{\neg A} \quad \text{Modus Tollens (MT)}$$

- Implementation:

```

1 rule('MT',                               % Name
2     [ FA --> FB,                          % Hypotheses
3       !FB ],
4     !FA,                                   % Deduction
5     [ 'Premisa'(1),                       % Proof
6       'Premisa'(2),
7       'Supuesto'(FA),
8       'E' --> (1,3),
9       'I' and (4,2),
10      'I' --> (3,5),
11      'I' ! (6) ]).
```

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```

- Generated proof:

```

MT: T[A --> B, !B] |- !A

1  A --> B           Premisa(1)
2  !B               Premisa(2)
3      A            Supuesto(A)
4      B            E-->(1,3)
5      B and!B     I and(4,2)
6  A --> B and!B   I-->(3,5)
7  !A              I!6
                  ok
```

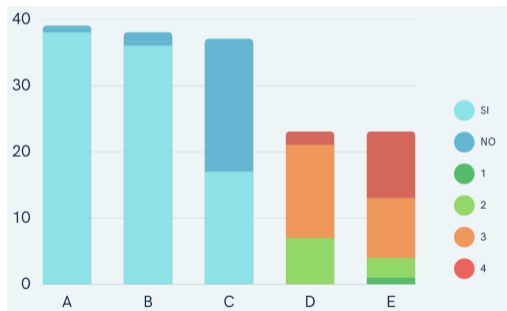
Natural Deduction: Home work (solve with/without derived rules)

$$\mathcal{T}[r \rightarrow q \wedge s, \neg(q \wedge s)] \vdash \neg r$$



Natural Deduction: Pilot experience during the 22/23 academic year.

- Small sample population but promising results.



- Please answer Yes or No:
- A: Have you used the program?
 - B: Have you used the program to check manifestations?
 - C: Have you used the program as a study tool?
- Rate from 1 (bad) to 4 (very good):
- D: Ease of use.
 - E: Usefulness for learning Natural Deduction.

Figure: Results of the DeduccionNatural.pl satisfaction survey on the Artificial Intelligence degree.

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<http://platon.etsii.urjc.es/~jarias/mmm/MDS/>

- Final Degree Projects:

- “MMDect: Metamorphic Malware Detection using Logic Programming” by Luciana.

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